# STUDIES IN POINT COLLOCATION METHOD FOR COMPUTING STRESS INTENSITY FACTOR

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By

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MARCH, 1985

# CERTIFICATE

CERTIFIED that the thesis titled, " STUDIES IN POINT COLLOCATION METHOD FOR COMPUTING STRESS INTENSITY FACTOR" has been submitted by Sri K. Sundararaju under my supervision and that this work has not been submitted elsewhere for award of a degree.

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## SYMBOLS

| | |
|---|---|
| $a$ | crack length in single edge notch specimen, cm |
| | half crack length in centre crack problem, cm |
| | half crack length in weld problem, in. |
| $a_n$, $d_n$ | coefficients of William's stress function |
| $E$ | Young's modulus, kgf/cm$^2$ |
| $K_I$ | opening mode stress intensity factor, ksc$\sqrt{cm}$ |
| $K_{II}$ | sliding mode stress intensity factor, ksc $\sqrt{cm}$ |
| $K_{III}$ | bearing mode stress intensity factor, ksc $\sqrt{cm}$ |
| $K_N$ | elastic stress concentration factor |
| $\Delta k$ | change in stress intensity factor, ksc $\sqrt{cm}$ |
| $r, \theta$ | angular position co-ordinates referred to crack tip |
| $H$ | height of the plate from the crack plane, cm |
| $W$ | specimen width, cm |
| $V_o$ | uniform displacement in 'y' direction applied to specimen, cm |
| $\sigma_o$ | uniform tensile stress applied to specimen, kgf/cm$^2$ |
| $x, y$ | co-ordinate axes with origin as crack tip, parallel and perpendicular, respectively, to crack plane |
| $\sigma_x, \sigma_y, \tau_{xy}$ | stress in x - and y - directions, kgf/cm$^2$ |

| | |
|---|---|
| $U_r$ , $U_\theta$ | disolacements r - and θ - directions, cm |
| $U_x$ , $U_y$ | displacements x - and y - directions, cm |
| $\Psi$ | William's stress function |
| $\Psi_e$ , $\Psi_o$ | odd and even parts of stress function |
| $\Psi_1$ | harmonic function |
| $\mu$ | shear modulus, $kgf/cm^2$ |
| $\nu$ | Poisson's ratio |
| $\gamma$ | gamma whose value is equal to $\nu/1 + \nu$ |
| n | number of boundary points |
| k | number of arbitrary constants of the stress function such that $k \leq 2n$ |
| R | radius of curvature of the butt weld profile,in fictitious crack tip |

## ABSTRACT

A boundary point collocation procedure is applied to find stress intensity factors of single edge crack problems and centre crack problems with traction and/or displacement prescribed boundary conditions. For solving the crack problems we made use of William's stress function [4] which gives the solution in terms of trigonametric terms with undetermined coefficients. The boundary conditions are derived from William's stress function [4] by successive partial differentiation. Thus in our present work we make use of direct boundary conditions, for collocation procedure. The usage of William's stress function prescribed for edge cracks is extended to centre crack problems as well in our work. The results thus, obtained are compared with the existing results and good agreement is observed. A butt weld improper penetration problem is idealized as a centre crack problem and its stress intensity factor is also calculated.

CHAPTER - 1

INTRODUCTION

## 1.1 General Introduction:

Fracture mechanics is a discipline of engineering dealing with the analysis and prediction of mechanical and structural behaviour of a cracked structure when it is loaded. Many structures may be made essentially crack (or flaw) free. However, the service conditions for many complex and high performance structures (air planes, offshore drilling rigs, bridges etc.) involve repeated loading, which often limits the life of a structure. The finite life is a result of repeated loads (and/or temperature fluctuations) which induce tiny cracks formed by the process called fatigue, often at some geometric discontinuity (for example, a hole). In service those tiny cracks grow leading to a cracked structure.

Many structures are typically fabricated by welding. Common weld designs often introduce geometries that are effective cracks. For example, in butt-welding of thick plates due to improper penetration, we get

a structure with an effective crack at the centre. Further the process of welding in itself, inevitably leads to a compromise between slow welding speed which minimizes the disturbance of the metal, and a high welding speed to increase productivity. The result is a structure that is unintentionally cracked by the time that it enters service.

Past experience shows that whenever unexpected failures (at design conditions) have occurred they always involve a flaw or a crack. Since it may not be possible to prevent cracks occurring in service it is necessary to have a means of assessing their effect under working condition in a given structure. It can well be stated here that a cracked structure "need not necessarily" have failed. Certainly it is true that some cracked structures have failed for all practical purposes. However, it is equally true that many large complex structures contain 'cracks' and inspite of them will provide satisfactory, safe and economical performance for many years to come. There is no implication here that cracks or flaws are desirable or intentionally introduced into structures. There is only the recognition that many structures are unintentionally flawed and we must deal with this reality. This justifies the investment of money, time and effort on the study of cracked structures.

Fracture mechanics provides a basis for quantitying the behaviour of cracks during both the propogation phase and the final failure. The important parameter in this field of science is the " Stress Intensity Factor", 'K', which is a measure of the magnitude of the stress occurring in the immediate vicinity of the tip of a crack. The stress intensity factor is a function of the loading on the cracked configuration and of the size and shape of the crack and other geometrical boundaries. Thus, there is a need to know, K, for cracked configurations which might be encountered in practice.

## 1.2 Literature Survey:

Exact mathematical solutions for stress intensity factors have been determined by sophisticated mathematical techniques, like perturbation techniques and the method of integral transforms, for a number of idealised geometric configurations and loading conditions. But in most complex engineering situations where it is difficult to obtain theoretical solutions. Various numerical methods must be used to obtain approximate solution. The two most general numerical methods for the elastic analysis of cracked solids are boundary collocation procedure and the finite element method. Finite element method is the most general of the two in that it can be applied to curved cracks and comples geometries, and

loading conditions. Collocation procedure can be applied to straight cracks and fairly simple external boundaries. However, for classes where a boundary collocation procedure is applicable, it offers the distinct advantage over the finite element method of greater accuracy, less input preparation and a relatively smaller system of simultaneous equations to be solved.

The boundary collocation procedure can be formally defined as follows: using a stress function, governing the stress field of a cracked solid, the boundary conditions are satisfied at a finite number of discrete boundary points on the outer boundary, of the cracked body. This is done by truncating the terms of the stress function such that the number of arbitrary constants of the stress function is equal to the number of discrete boundary conditions. The values of the constants are chosen such that the boundary conditions are satisfied exactly at the discrete boundary points.

W.K. Wilson [1] solved a plane elasticity problem of a finite rectangular plate having a centre crack with arbitrary orientation under tensile loading, by means of an Airy's stress function. He reduced the boundary conditions into one of the three following forms:

(a) surface tractions

(b) the normal and tangential derivative of the stress function of the boundary points,

(c) the Airy's stress function itself, and its normal derivative of the boundary points.

He observed that boundary collocation method using the boundary conditions in (c) gives the most accurate results. He also claims that stability, simplicity and relative independence of results w.r.t. the position of boundary points are the method's main features.

M. Isida [2] studied a centre crack problem with the crack exactly perpendicular to the loading conditions using a complex stress function. J.E. Snawley [3] analysed an edge crack problem in tensile loading using a real stress function formulated by M.L. Williams [4]. Both Isida and Srawley used the boundary conditions for collocation procedure in the form as given in (c) and they obtained very good results.

1.3 Objective and Scope of Present Work:

It may well be stated here that when we have both the displacement and traction prescribed problems over the boundary it will be convenient to use them directly as they are, with some special technique to achieve stability and accuracy even at the expense of relatively higher computational time. This is not

unjustifiable considering the advent of computational machines in the present days.

An attempt is made in our present work to use the boundary conditions involving second order partial differentiation of the William's stress function, for both edge crack and centre crack problems. By this we incorporate the traction and displacement boundary conditions directly in our problems. The centre crack problem is analysed treating it as two edge crack problems. Also a butt weld improper penetration problem is idealized as a centre crack problem and is analysed using the boundary collocation procedure.

In the chapter [2] we describe the formulation of the problems with the necessary boundary conditions. The required equations in this connection are derived. In chapter [3] we present the method of solution along with results and discussions.

# CHAPTER - 2

# CRACKS UNDER TENSILE LOADING

## 2.1 Introduction:

In this chapter William's stress function [4] governing the stress field around the crack tip of an edge crack is reviewed, along with the general stress field around a crack tip and the modes of deformation. The relationship between the stress intensity factor and the first coefficient of William's stress function is derived. William's stress function is used to analyse edge crack problems with traction and displacement boundary conditions. A centre crack problem is analysed treating it as two edge crack problems. A butt weld problem with improper penetration is treated a centre crack problem and the necessary equation in this regard are also derived.

## 2.2 Stress Field Near a Crack Tip:

The linear elastic solution for the stress field around a crack shows that the stress components $\sigma_{ij,s}$ at a distance 'r' from the crack tip can be expressed in the following form (fig. 2.1.a)

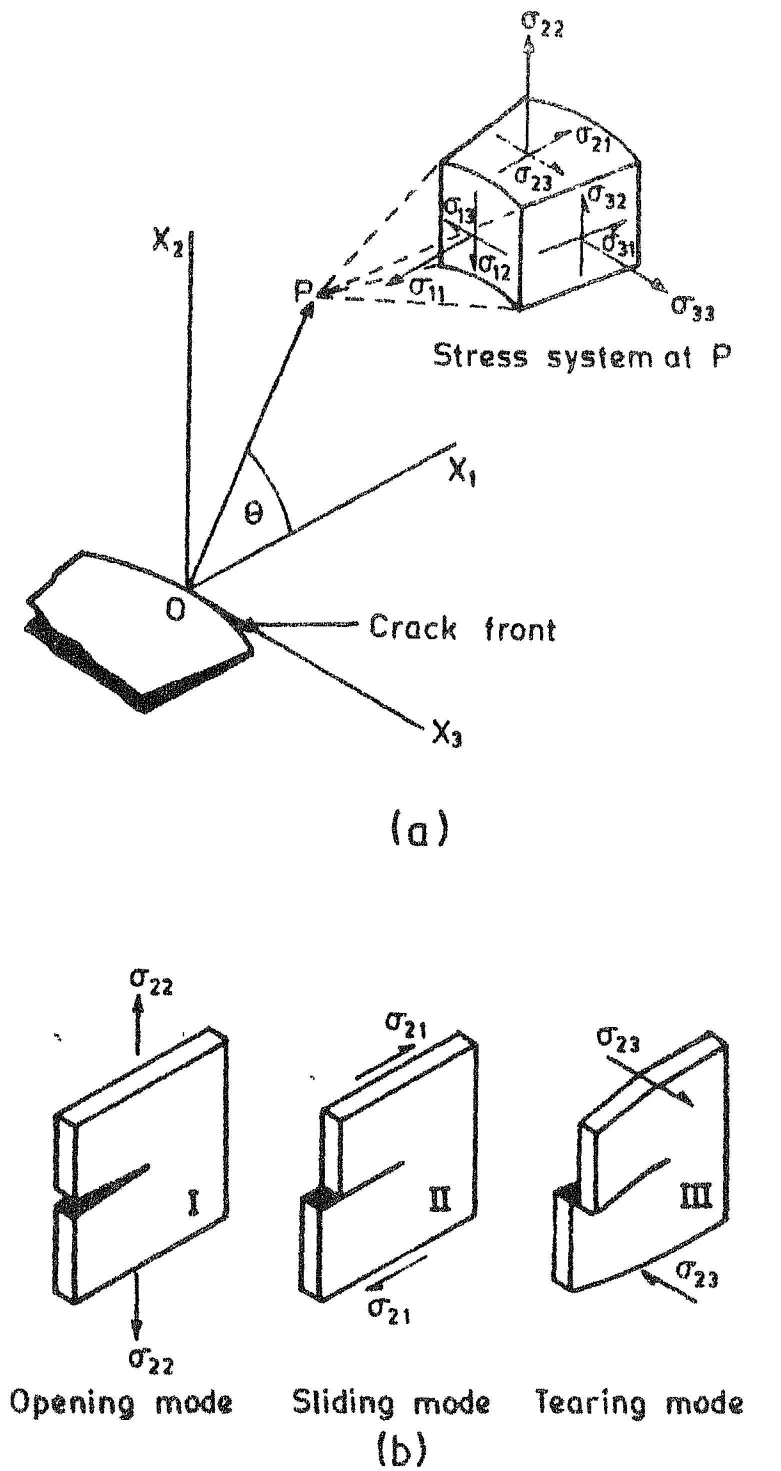


Fig. 2.1 (a) Crack co-ordinate system
(b) Modes of deformation

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta) + \text{'other terms'} \qquad (2.1)$$

where K, is the stress intensity factor and $(r, \theta)$ are the polar co-ordinates. If the point $(r, \theta)$ is sufficiently close to the tip, i.e., $r \ll$ crack length, the 'other terms' in the above equation are negligible compared to the first term. The above elastic solution predicts infinite stresses at the crack tip ($r = 0$), which can not occur in practice, since plastic flow takes place in the highly stressed region near the crack tip. However, if the region of plastic flow is small compared to the region over which $\sqrt{r}$ term dominates the stress field, it may be assumed that, the behaviour of the crack is determined by the elastic stress intensity factor. This assumption forms the basis of linear elastic fracture mechanics.

## 2.3 Modes of Deformation:

The stress intensity factor, K, is a function of the loading on the cracked configuration and the size and shape of the crack and other geometrical boundaries. It has the dimensions of stress $\times \sqrt{\text{length}}$. There are three distinct types of cracking modes which are characterized by different symbols for the stress intensity factors. These are illustrated in fig.(2.1.b). Mode I

characterized by $K_I$ is known as the opening mode. Mode II characterized by $K_{II}$ is known as the sliding mode and mode III characterized by $K_{III}$ is known as the tearing mode. The stress intensity factor for each mode can be formally defined as follows:

$$K_I = \lim_{r \to 0} \left\{ \sqrt{2 \pi r}\ \sigma_{22}(r,0) \right\}$$

$$K_{II} = \lim_{r \to 0} \left\{ \sqrt{2 \pi r}\ \sigma_{21}(r,0) \right\}$$

$$K_{III} = \lim_{r \to 0} \left\{ \sqrt{2 \pi r}\ \sigma_{23}(r,0) \right\} \quad \ldots \quad (2.2)$$

out of these three, the opening mode, is the most common mode found in practical situations and in our present work we restrict ourselves to determine the stress intensity for the opening mode.

## 2.4 William's Stress Function:

In his paper M.L. Williams [4] has formulated a stress function in polar co-ordinates governing the stress field around the tip of an edge crack. Besides satisfying the biharmonic equation the function also satisfies the stress free conditions along the crack edges. The function $\Psi$, can be written in terms of polar co-ordinates (fig. 2.2) as follows:

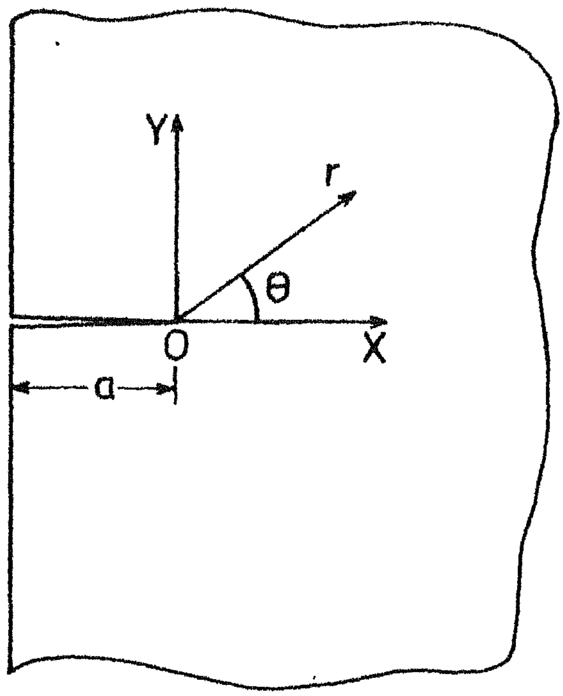


Fig. 2.2 An edge crack in an infinite plate and the co-ordinate system.

$$\Psi = \Psi_e + \Psi_o$$

where $\Psi_e$ and $\Psi_o$ are even and odd parts of the stress function given by

$$\Psi_e = \sum_{n=1,2,..\infty} \Big[ (-1)^{n-1} d_{2n-1}\, r^{n+\frac{1}{2}} \Big\{ -\cos\left(n - \tfrac{3}{2}\right)\theta + \frac{2n-3}{2n+1} \cdot \cos\left(n + 1/2\right)\theta \Big\} \quad (2.3)$$
$$+ (-1)^n d_{2n} \cdot r^n \Big\{ -\cos(n-1)\theta + \cos(n+1)\theta \Big\} \Big]$$

and

$$\Psi_o(r,\theta) = \sum_{n=1,2,..\infty} \Big\{ (-1)^{n-1} a_{2n-1}\, r^{n+1/2} [\sin(n - 3/2)\theta - \sin(n + 1/2)\theta]$$
$$+ (-1)^n a_{2n} \left[ -\sin(n-1)\theta + \frac{n-1}{n+1} \sin(n+1)\theta \right] \Big\} \quad ...(2.4)$$

The constants $d_n$'s and $a_n$'s depend upon the boundary conditions, more specifically either upon boundary conditions, at infinity in the case of infinite sector, or upon those at some fixed radius when the plate has finite dimensions.

## 2.5 Formulations of Stress and Displacement Equations:

Since the William's stress function is basically an Airy's stress function, the three basic stresses can

be derived by successive partial differentiation as follows:

$$\sigma_x = \frac{\partial^2 \Psi}{\partial y^2} \quad \sigma_y = \frac{\partial^2 \Psi}{\partial x^2} ; \quad \tau_{xy} = -\frac{\partial^2 \Psi}{\partial x \partial y} \quad \ldots \quad (2.5)$$

In matrix notation these equations can be written in terms of polar co-ordinates as follows,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \sin^2\theta & \frac{\sin 2\theta}{r} & \frac{\cos^2\theta}{r} & -\frac{\sin 2\theta}{r^2} & \frac{\cos^2\theta}{r^2} \\ \cos^2\theta & -\frac{\sin 2\theta}{r} & \frac{\sin^2\theta}{r} & \frac{\sin 2\theta}{r^2} & \frac{\sin^2\theta}{r^2} \\ -\sin\theta\cos\theta & -\frac{\cos 2\theta}{r} & \frac{\sin\theta\cos\theta}{r} & \frac{\cos 2\theta}{r^2} & \frac{\sin\theta\cos\theta}{r^2} \end{bmatrix} \times [\phi]$$

where $[\phi]^T = [\frac{\partial^2 \Psi}{\partial r^2} \quad \frac{\partial^2 \Psi}{\partial r \partial \theta} \quad \frac{\partial \Psi}{\partial r} \quad \frac{\partial \Psi}{\partial \theta} \quad \frac{\partial^2 \Psi}{\partial \theta^2}]$

The radial and circumferential displacements, $U_r$ and $U_\theta$ respectively, can be formulated in terms of the biharmonic stress function according to Coker and Filon [5] as follows:

$$2\mu\, U_r = -\frac{\partial \Psi}{\partial r} + (1 - \gamma)\, r \frac{\partial \Psi_1}{\partial \theta}$$

$$2\mu\, U_\theta = -\frac{1}{r}\frac{\partial \Psi}{\partial \theta} + (1 - \gamma)\, r^2 \frac{\partial \Psi_1}{\partial r} \quad (2.7)$$

where $\mu$ = shear modulus

$\gamma = \nu/1+\nu$ where $\nu$ is the Poisson's ratio

and $\Psi_1$ is a harmonic function and is related to the stress function $\Psi$ as follows:

$$\nabla^2\Psi = \frac{\partial}{\partial r}\left(r\frac{\partial\Psi_1}{\partial\theta}\right) ; \qquad (2.8)$$

Making use of the above equations (2.7) and (2.8) the displacement field equations, $U_r$ and $U_\theta$ can be found out easily. To facilitate applications of boundary conditions $U_r$ and $U_\theta$ can be transformed to the cartesian co-ordinates using the following relationship (fig. 2.3).

$$U_x = U_r \cos\theta - U_\theta \sin\theta$$
$$U_y = U_r \sin\theta + U_\theta \cos\theta \qquad (2.9)$$

where '$\theta$' is the angle between the cartesian and polar co-ordinate system.

## 2.6 Relationship Between Stress Intensity, $K_I$, Factor, and the First Coefficient of William's Stress Function, $d_1$:

The stress intensity factor $K_I$, may be derived in terms of the first coefficient of the William's stress function $d_1$, as follows:

From the equation (2.3) it is obvious that the " other terms" vanishes at the crack tip i.e. when $r = 0$. Hence, taking only the first term, we get

$$\sigma_y = \frac{\partial^2\Psi_e}{\partial x^2} = \frac{-d_1}{r}\,\frac{\cos\theta}{2}\left(1 + \frac{\sin\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \ldots \quad (2.10)$$

The expression for '$\sigma_y$', given in literature based on Westergaard [6] is as follows:

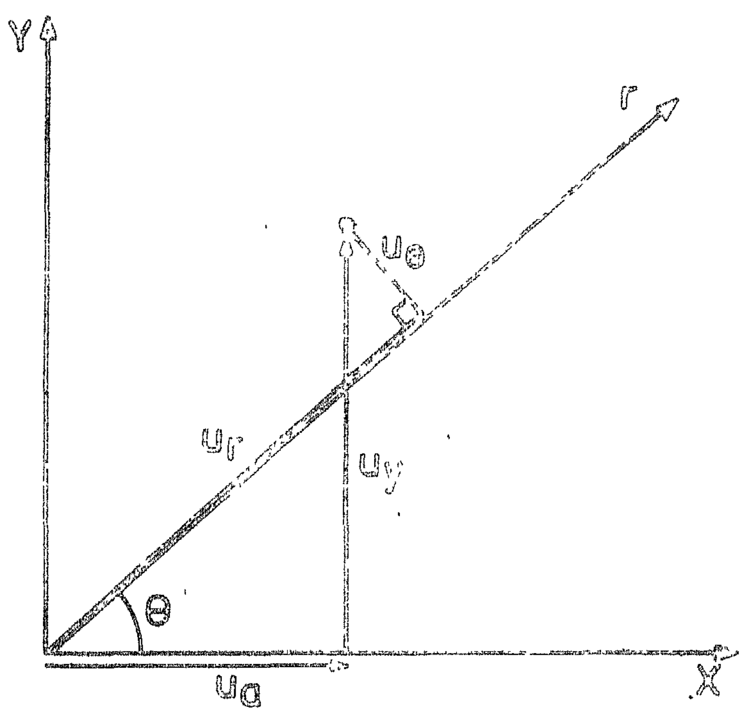


Fig. 2.3 Transformation representation of displacement.

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \frac{\sin\theta}{2}\sin\frac{3\theta}{2}\right) \qquad (2.11)$$

comparing equation (2.10) and (2.11) we find

$$K_I = -\sqrt{2\pi} \cdot d_1 \qquad (2.12)$$

This relationship gives us a method that if by some means the constants $d'_s$ are found out for a particular problem then $K_I$ can be determined from $d_1$. The reason for not considering the odd series of the stress function $\Psi_o$ will be given later along with the method of solutions.

2.7 Single Edge Crack Specimen:

The single edge notch specimen (fig. 2.4a) has been analysed by Sraweley et al[3] using the stress function $\Psi$, and its normal derivative $\partial\Psi/\partial n$, as the boundary conditions and their results compared well with the results obtained by other methods. It is to be noted that it was possible for Sraweley to use the derived boundary conditions ($\Psi$ and $\partial\Psi/\partial n$) as the loading and geometry of the specimen were fairly simple. The boundary conditions used by Srawley are as follows (fig.2.4a)

Along A-B

$$\Psi = 0 \; ; \; \frac{\partial\Psi}{\partial x} = 0 \; ;$$

Along B-C

$$\Psi = \sigma_o \left(\frac{x+a}{r_2}\right)^2 \; ; \; \frac{\partial\Psi}{\partial y} = 0 \; ; \qquad (2.13)$$

contd...

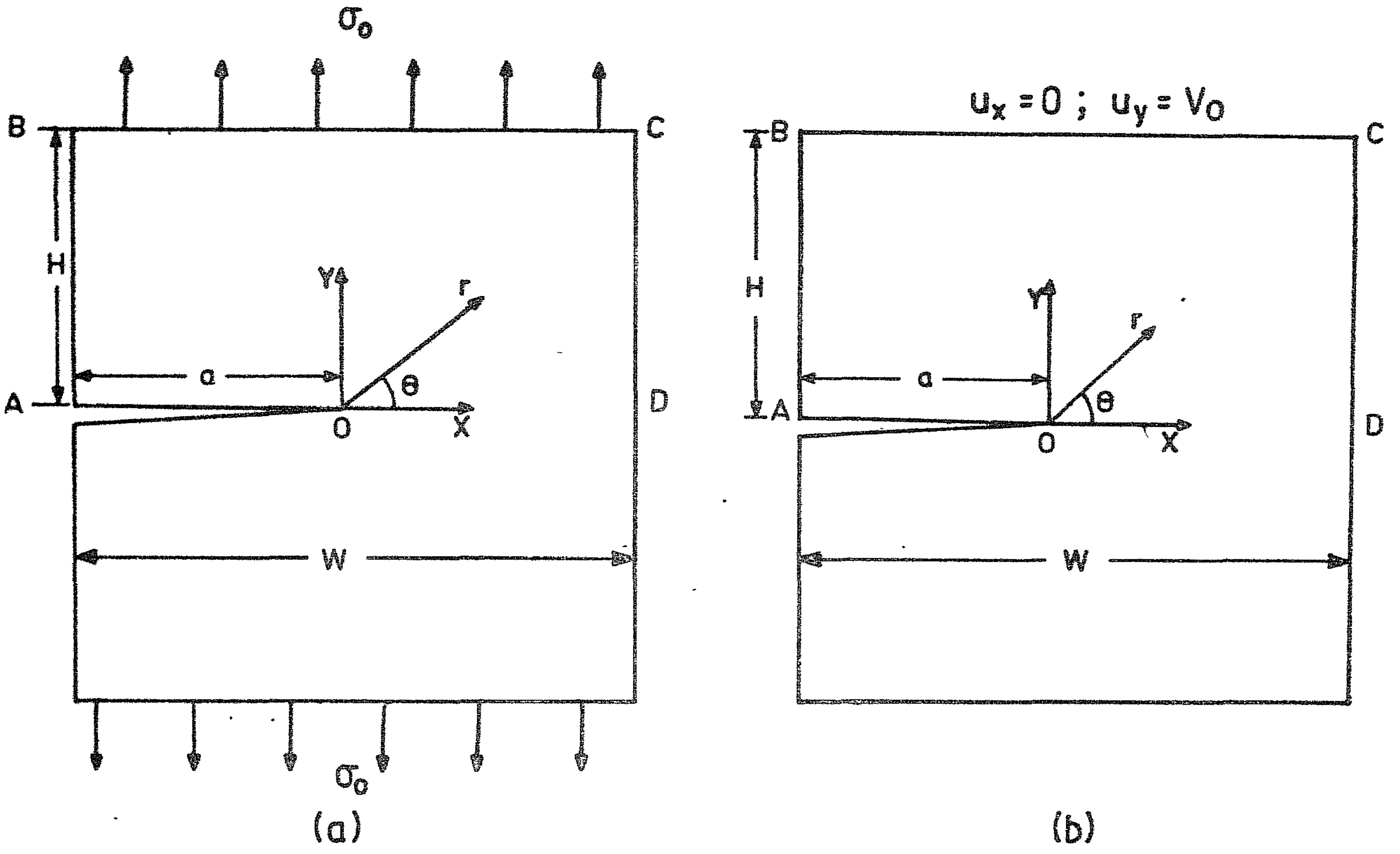


Fig. 2.4 (a) Single edge crack under uniform tensile loading (b) Single edge notch under uniform displacement.

a - Cracklength ; 2H - Height of the plate ; W - Width of the plate
$V_o$ - Displacement in 'Y' direction ; $\sigma_o$ - The uniform tensile stress.

Along C-D

$$\Psi = \frac{\sigma_o W^2}{2} ; \frac{\partial \Psi}{\partial x} = \sigma_o W ; \qquad (2.13)$$

where a, and W are the crack length and width of the plate and $\sigma_o$ is the uniform tensile stress applied along B-C.

In our work, in an attempt to use the traction, and displacement boundary conditions involving second order partial differentiation, the following direct boundary conditions are used.

Along A-B

$$\sigma_x = 0 \; ; \quad \tau_{xy} = 0 \; ;$$

Along B-C

$$\sigma_y = \sigma_o \; ; \quad \tau_{xy} = 0 \; ; \qquad (2.14)$$

Along C-D

$$\sigma_x = 0 \; ; \quad \tau_{xy} = 0 \; ;$$

As can be observed from the figure there exists a symmetry with respect to the x-axis and hence only top half the specimen and hence the even part of the William's stress function need to be considered. Therefore

$$\Psi (r, \theta) = \Psi_e (r, \theta) \qquad (2.15)$$

This is true for all the cases where there is symmetry about x-axis and since in our present work we deal only problems of this kind, equation (2.15) is true for all

the cases that we consider. Also if displacement conditions are prescribed along the boundary B-C (fig. 2.4b) then the boundary conditions become as follows:

Along A-B

$$\sigma_x = 0 \quad ; \quad \tau_{xy} = 0 \; ;$$

Along B-C

$$U_x = 0 \; ; \quad U_y = v_o \quad ; \qquad (2.16)$$

Along C-D

$$\sigma_x = 0 \; ; \quad \tau_{xy} = 0 \quad ;$$

A square specimen of this kind is analysed by Kobayashi [7] using finite element techniques.

## 2.8 Centre Crack Problem:

The centre crack problem is treated as two edge crack problems which are separated by the mid plane L-L' (fig. 2.5a) as shown. We consider only one quarter of the specimen because of symmetry, with the following boundary conditions (fig. 2.5b)

Along A-B

$$U_x = 0 \quad ; \quad \tau_{xy} = 0 \; ;$$

Along B-C

$$\sigma_y = \sigma_o \; ; \quad \tau_{xy} = 0 \; ; \qquad (2.17)$$

Along C-D

$$\sigma_x = 0 \quad ; \quad \tau_{xy} = 0$$

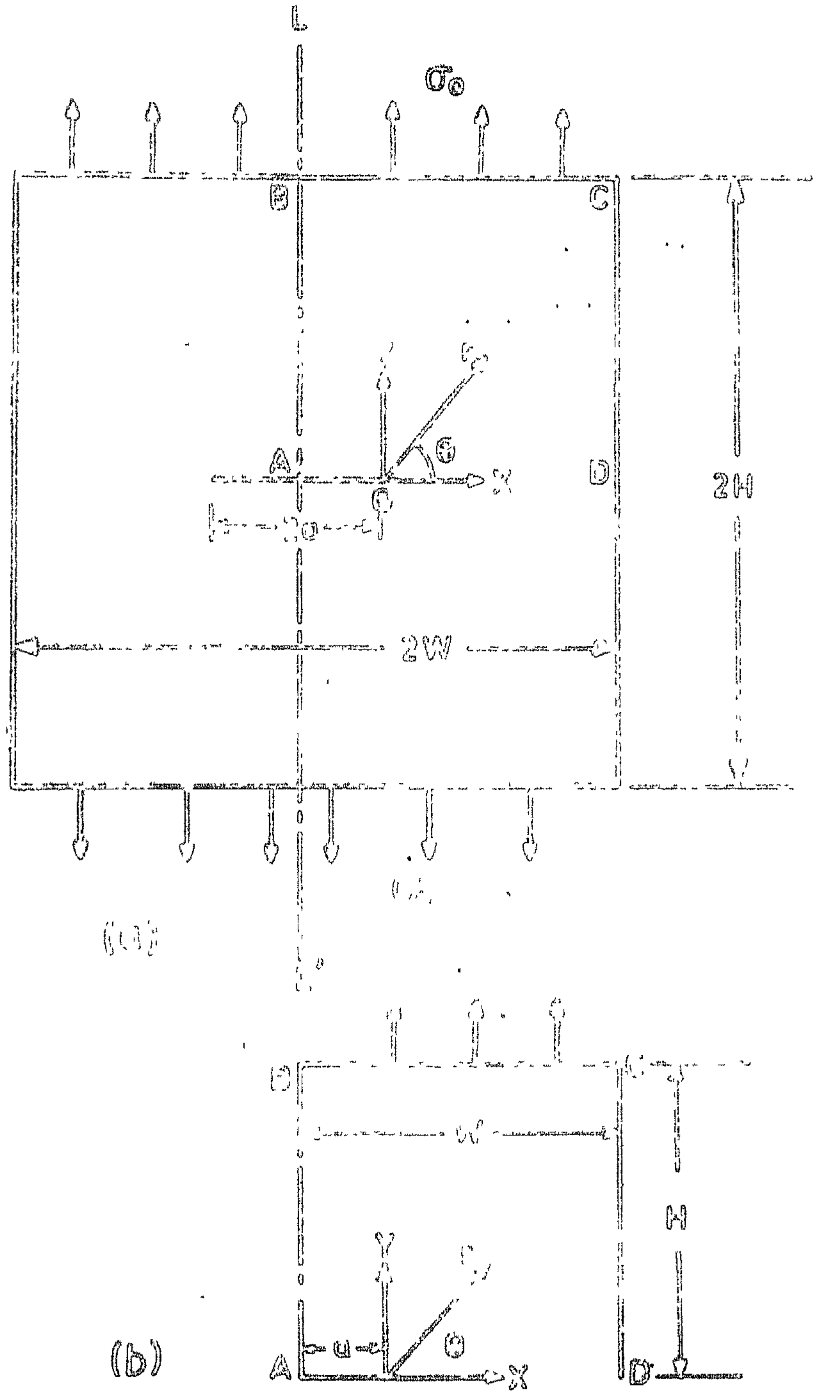


**Fig. 2.5 (a) Centre crack problem (b) One quarter of the specimen for analysis purpose.**

This specimen is analysed by M. Isida and others [2] using a complex stress function prescribed for centre cracks. In our work we use the traction and displacement boundary conditions directly instead of $\Psi$ and $\partial \Psi / \partial n$ as used by Isida. We use the same William's stress function which is prescribed for edge cracks, for this centre crack problem incorporating the displacement boundary condition. Thus the usage of William's stress function is extended to the centre crack problems as well in our work.

## 2.9 Problem of Butt-Weld with Improper Penetration:

Weld defects constitute a group of stress raisers which have more serious effects than reinforcement or undercuts, of the weld profile. One category of such defects covers those caused by insufficient penetration in butt-welds. The American welding society [8] recommends that the butt-welds with partial joint preparation be used for static loading only. According to their tests full penetration butt-welds on plates (square ground welds) are only possible on plates $1/8^{th}$ of an inch. With a root opening of one half the plate thickness, full penetration is obtainable upto 1/4 inch thick, but the root of the first weld must be chipped out to sound metal before depositing the second weld. For economical reasons, these recommendations can not

always be adhered to and butt welds with improper penetration are common in non-critical welded connections subjected to cyclic loads. The economical need is felt for standards that specify permissible defect sizes in welded metal structures. Lack of penetration is assumed to be similar to a crack at the interface of two plates joined by welding (fig. 2.6a). Using the stress intensity factor for this geometry and loading a very good approximation can be made for the stress field and then we can determine whether a joint will perform to our expectations.

The boundary conditions for this geometry (fig. 2.6b) is as follows:

Along A-B

$$U_x = 0 \; ; \quad \tau_{xy} = 0 \; ;$$

Along B-C

$$\sigma_y = \sigma_o \; ; \; \tau_{xy} = 0 \; ; \qquad (2.18)$$

Along C-D

$$\tau_{xy} = 0 \; ; \quad \sigma_x = 0 \; ;$$

Along DEF

$$\sigma_n = 0 \; ; \quad \tau_s = 0 \; ;$$

where $\sigma_n$ and $\tau_s$ are normal and shear stresses along the curved boundary which is circular with radius R DEF, and are related to $\sigma_x$, $\sigma_y$, $\tau_{xy}$ stresses as

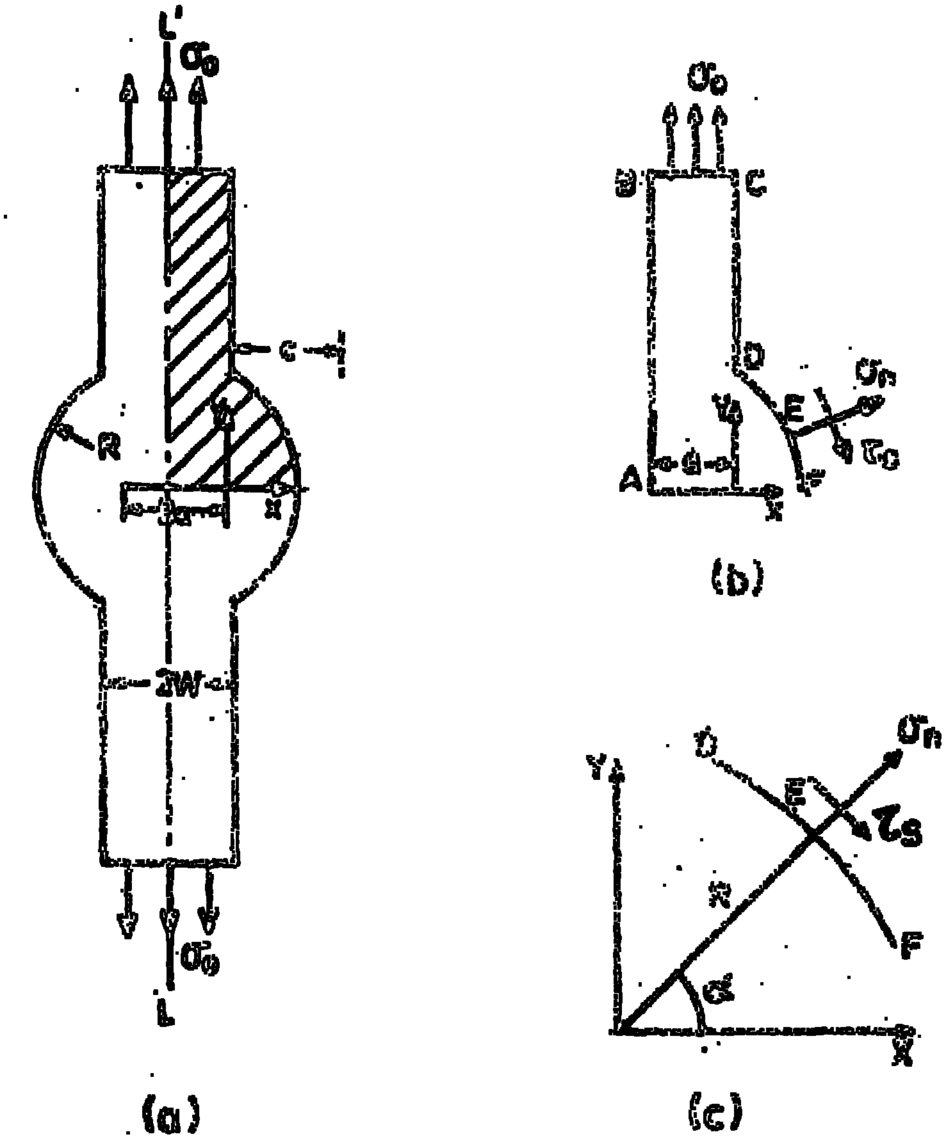


Fig. 2.8 (a) Butt-weld improper penetration problem.
(b) One quarter of the butt-weld problem for analysis purpose.
(c) Co-ordinate representation of stress transformations for the curved boundary 'DEF'.

follows (fig. 2.6c)

$$\sigma_n = \sigma_x \cos^2\alpha + 2\tau_{xy} \sin\alpha \cos\alpha + \sigma_y \sin^2\alpha$$

$$\tau_s = (\sigma_x - \sigma_y) \sin\alpha \cos\alpha - \tau_{xy} (\cos^2\alpha - \sin^2\alpha) \quad \ldots \quad 2.19$$

and $\alpha$ is as shown in (fig. 2.6c).

We discuss the method of solution and the results for all the problems formulated, in the next chapter.

# CHAPTER - 3

## RESULTS AND DISCUSSION

### 3.1 Introduction:

In this chapter we describe the method of solution and the results in detail as applied to single edge-crack problems and centre crack problems. The results obtained agreed well with the existing results only upto H/W ratio of 0.6, thus necessitating the need to improve the solution procedure. An improvement in the solution procedure is suggested in the present work, and is applied to all the problems discussed in sections (2.7, (2.8) and (2.9). Reasonably good results were obtained even upto H/W ratio of 1.5. The results are presented in a normalized form.

### 3.2 Solution Procedure and Discussion of Results:

If we consider the single edge notch problems and centre crack problems as discussed in sections (2.7), (2.8) and (2.9), we observe that, at each point over the boundary of the specimens we have two boundary conditions to be satisfied as given by equations 2.14, 2.16, 2.17 and 2.18.

Since we have to take only the even series of the stress function, $\Psi_e$ , as explained earlier, because of symmetry, we get exactly '2n' constants for each 'n', in the respective expressions of the boundary conditions. Thus to solve the problem we truncate the expressions with '2n' arbitrary constants ($d_n$'s) and satisfy the boundary conditions at 'n' number of points selected over the boundary at equal distances. We get exactly '2n' equations in '2n' constants. Now if we solve this system of linear equations we can find stress intensity factor, $K_I$ , using the relation 2.12.

3.2.1 Selection of Boundary Points:

A subroutine is developed which would give the co-ordinates of the required 'n' number of points over the boundary say A.B.C.D. in the case of a single-edge notch problem (fig. 2.4a) distributed at equal distances along the periphery. The input parameters to this routine are, the crack length, a, width of the plate 'W', the height of the plate from the crack plane H, and of course, the required number of points, 'n'. Thus the geometry of the plate is taken care of the co-ordinates of the boundary points that subsequently enter main solution procedure. It can be stated here that the input preparation to the Collocation procedure is

very simple when compared to the finite element method.

3.2.2 Solution of Equations:

The boundary conditions are satisfied exactly at the 'n' number of points, i.e. the error is made zero at these points in Collocation procedure. Gaussian elimination is employed to solve the system of linear equations. We are interested in finding mainly, the first coefficient of the William's stress function $d_1$, which is to be used to compute the stress intensity factor, $K_I$. If we bring the constant $d_1$ to the last place of the truncated expressions Gaussian elimination eliminates the possibility of additional computations and thereby subsequent error in finding $d_1$.

As we are approximating the entire boundary points by a finite number of boundary points, n, it is expected that the exact results would be approached when 'n' tends to infinity. Also we approximate the stress field by using 'k' number of constants such that k = 2n. Reasonably good results are obtained within allowable error if the number of points 'n' increased until the coefficient $d_1$ reaches a stable value. This procedure is repeated for differqnt values of 'n' for a particular plate dimensions i.e. fixing a/W and H/W. The solutions for single edge crack

problem is presented in a tabular form in which the value of $d_1$ is shown against for each 'n' number of points taken over the boundary (table 3.a).

It is observed that this method of solution i.e. taking exactly the required number of points over the boundary to find the unknown coefficients d's does not give good convergence of $d_1$ when the ratio $H/W >$ 0.6; (table 3.a) therefore the need to improve the technique is felt and the following method is suggested.

We truncate the expressions of the boundary conditions to 'k' number of constants and select 'n' number of points over the boundary such that $2n > K$. That is we take more number of points than required to find the constants and treat the system of linear equations as over determinant. This kind of treatment of the system gives good convergence of the coefficient $d_1$ as both the number of points 'n' and constants 'k' are increased. The values of $d_1$ for different a/W and H/W ratios against the number of boundary points 'n' and arbitrary constants $d'_s$ are presented in table (3.b). We employed subroutine 'LLSQAR' available in our 'IMSL' routines. This method of solution gives good results even upto H/W ratio of 1.5, above which the stress intensity factor is assumed to be insensitive for all practical purposes as there is no appreciable change observed.

TABLE-3a

Results obtained by solving (2n x 2n) system for single-edge-crack problem with traction prescribed boundary conditions

| Trial No. | a/W | H/W | No. of points over the boundary n | No. of cons-tants 'k' | First co-efficient $d_1$ |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 0.5 | 0.4 | 18 | 36 | - 165.83 |
| 2 | 0.5 | 0.4 | 21 | 42 | - 162.62 |
| 3 | 0.5 | 0.4 | 24 | 48 | - 163.87 |
| 4 | 0.5 | 0.4 | 27 | 54 | - 165.31 |
| 5 | 0.5 | 0.4 | 30 | 60 | - 164.95 |
| 1 | 0.5 | 0.5 | 18 | 36 | - 167.23 |
| 2 | 0.5 | 0.5 | 21 | 42 | - 134.1091 |
| 3 | 0.5 | 0.5 | 24 | 48 | - 150.56 |
| 4 | 0.5 | 0.5 | 27 | 54 | - 150.9178 |
| 5 | 0.5 | 0.5 | 30 | 60 | - 150.3091 |
| 1 | 0.5 | 0.6 | 18 | 36 | - 140.96 |
| 2 | 0.5 | 0.6 | 21 | 42 | - 125.21 |
| 3 | 0.5 | 0.6 | 24 | 48 | - 146.075 |
| 4 | 0.5 | 0.6 | 27 | 54 | - 145.29 |
| 5 | 0.5 | 0.6 | 30 | 60 | - 147.847 |
| 1 | 0.4 | 0.6 | 18 | 36 | - 94.213 |
| 2 | 0.4 | 0.6 | 21 | 42 | - 76.222 |
| 3 | 0.4 | 0.6 | 24 | 48 | - 99.15 |
| 4 | 0.4 | 0.6 | 27 | 54 | - 99.02 |
| 5 | 0.4 | 0.6 | 30 | 60 | - 101.43 |
| 1 | 0.4 | 0.7 | 18 | 36 | - 70.12 |
| 2 | 0.4 | 0.7 | 21 | 42 | - 93.101 |
| 3 | 0.4 | 0.7 | 24 | 48 | - 111.3 |
| 4 | 0.4 | 0.7 | 27 | 54 | - 103.6 |
| 5 | 0.4 | 0.7 | 30 | 60 | - 107.82 |

TABLE-3b

Results obtained by solving (2n x k) system for single edge crack problem with traction prescribed boundary conditions

| Trial No. | a/W | H/W | No. of points over the boundary n | No. of constants k | First coefficient $d_1$ |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| | 0.3 | 1.0 | 75 | 50 | - 63.216 |
| | | | 78 | 50 | - 62.823 |
| | | | 80 | 50 | - 63.140 |
| | | | 83 | 60 | - 63.165 |
| | | | 80 | 60 | - 62.99 |
| | | | 83 | 60 | - 62.300 |
| | 0.1 | 1.0 | 80 | 50 | - 93.96 |
| | | | 83 | 50 | - 93.93 |
| | | | 80 | 56 | - 94.13 |
| | | | 83 | 56 | - 94.14 |
| | 0.5 | 1.0 | 80 | 60 | - 141.26 |
| | | | 83 | 60 | - 141.06 |
| | 0.6 | 1.0 | 80 | 60 | - 219.09 |
| | | | 83 | 60 | - 220.99 |
| | 0.7 | 1.0 | 80 | 56 | - 258.91 |
| | | | 83 | 56 | - 376.82 |
| | | | 80 | 46 | - 314.75 |
| | | | 83 | 46 | - 315.82 |
| | | | 80 | 44 | - 295.02 |
| | | | 83 | 44 | - 292.28 |
| | 0.3 | 0.5 | 7[illegible] | 50 | - 64.75 |
| | | | 7[illegible] | 50 | - 64.88 |
| | | | 75 | 60 | - 64.89 |
| | | | 78 | 60 | - 64.948 |
| | | | 80 | 60 | - 64.91 |
| | | | 83 | 60 | - 64.80 |
| | 0.4 | 0.5 | 75 | 50 | - 95.340 |
| | | | 78 | 50 | - 95.322 |

Contd.....

Contd.....<u>Table-3b</u>



| 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| | 0.5 | 0.75 | 75 | 50 | - 142.04 |
| | | | 78 | 50 | - 142.04 |
| | 0.6 | 0.75 | 75 | 50 | - 221.25 |
| | | | 78 | 50 | - 221.25 |
| | 0.7 | 0.75 | 75 | 50 | - 374.51 |
| | | | 78 | 50 | - 368.57 |
| | | | 80 | 50 | - 375.6 |
| | | | 83 | 50 | - 368.89 |
| | | | 80 | 60 | - 377.05 |
| | | | 83 | 60 | - 370.07 |
| | | | 80 | 46 | - 369.70 |
| | | | 83 | 46 | - 369.163 |
| | 0.3 | 0.5 | 75 | 40 | - 70.96 |
| | | | 78 | 40 | - 71.19 |
| | | | 75 | 50 | - 71.52 |
| | | | 78 | 50 | - 71.48 |
| | | | 75 | 60 | - 71.63 |
| | | | 78 | 60 | - 71.51 |
| | 0.4 | 0.5 | 75 | 60 | - 104.02 |
| | | | 78 | 60 | - 103.93 |
| | | | 80 | 60 | - 103.90 |
| | | | 83 | 60 | - 103.95 |
| | 0.5 | 0.5 | 80 | 60 | - 150.40 |
| | | | 83 | 60 | - 150.42 |
| | | | 80 | 64 | - 150.41 |
| | 0.6 | 0.5 | 75 | 60 | - 227.19 |
| | | | 78 | 60 | - 227.11 |
| | 0.7 | 0.5 | 75 | 60 | - 377.52 |
| | | | 78 | 60 | - 377.99 |
| | 0.2 | 0.5 | 75 | 60 | - 46.68 |
| | | | 78 | 60 | - 46.52 |

The stress intensity factor is presented as a dimensionless ratio $K_I/K_0$ against a normalized crack length a/W, fig. (3.1). The normalizing parameter $K_0$ has the dimensions of a stress intensity factor and is so chosen as to emphasize some important aspect of the solution, e.g, to emphasize the effect of additional boundaries, $K_0$ would be equal to $K_3$ in the absence of such boundaries [9]. This form of presentation shows clearly the effects of important features such as geometrical boundaries and stress distributions.

The improved method of solution is applied to the problem discussed in sections 2.7 to 2.9 and their results are plotted in the normalized form as shown in figs. (3.2), (3.3) and (3.4). The convergence of the coefficient $d_1$ against number of boundary points 'n' and the number of constants 'k' for different a/W and H/W ratios are also presented in tables (3.c) (3.d) and (3.e). A square specimen of single edge crack with displacement prescribed boundary conditions (section 2.7) has been analysed by Kobayashi [~~9~~7] et al, using finite element method. In our present we studied the specimen with three different H/W ratios of 0.5, 0.75 and 1.0. For H/W ratio of 0.5 our results compared well with those obtained by finite element method for a/W > 0.2. The maximum error limit when compared with

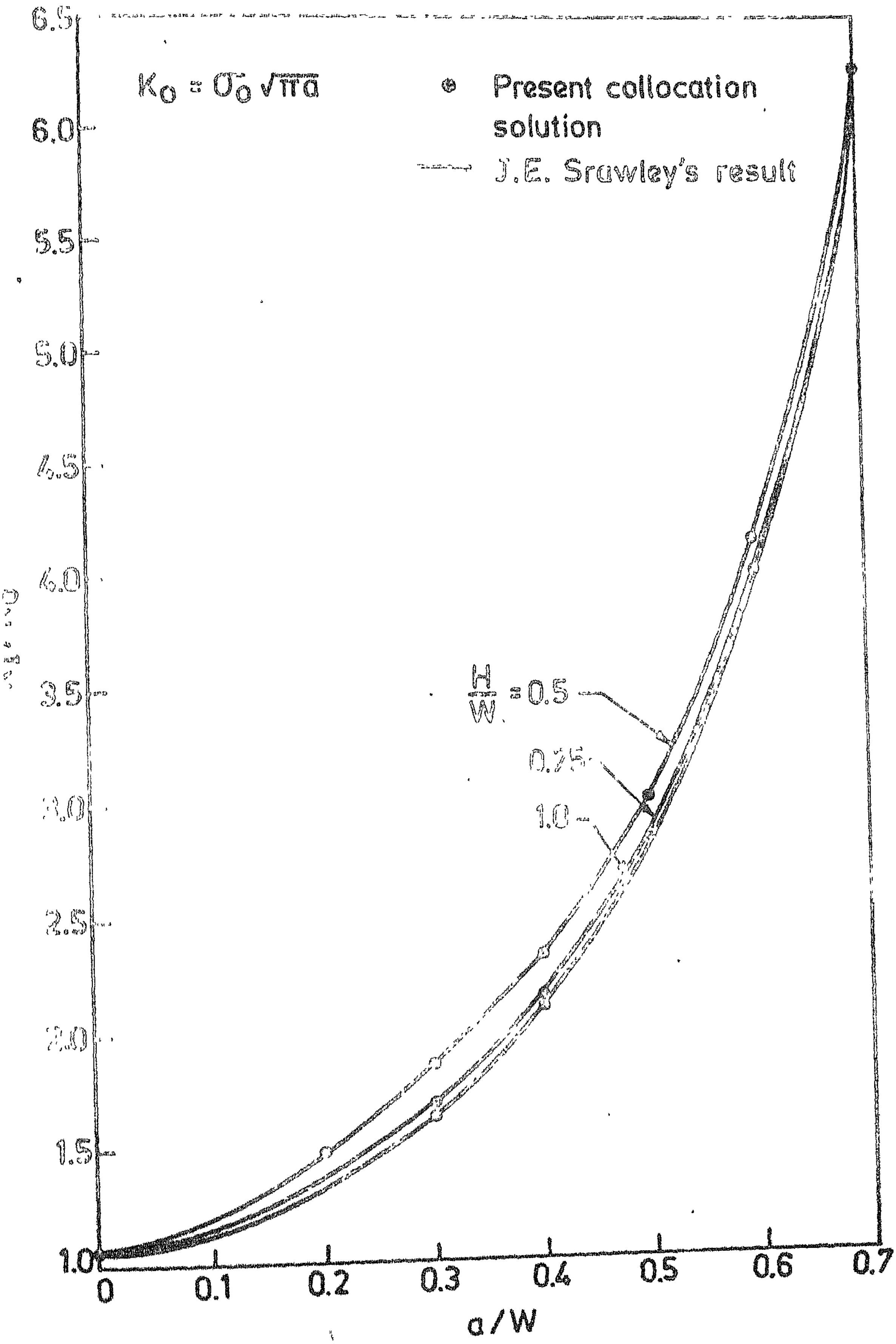


Fig. 3.1 Edge crack traction prescribed specimen.

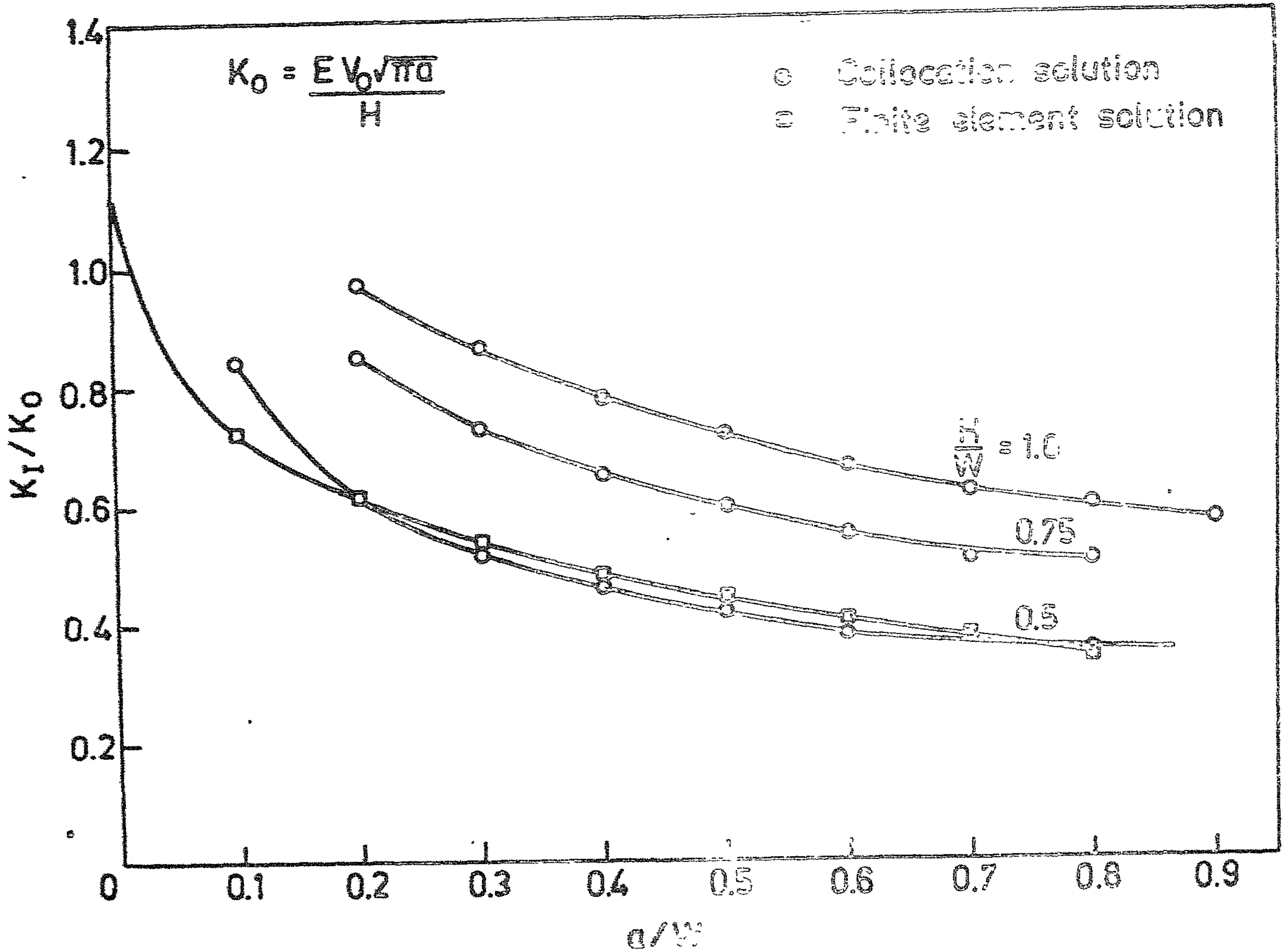


Fig. 3.2 Edge crack displ... prescribed specimen.

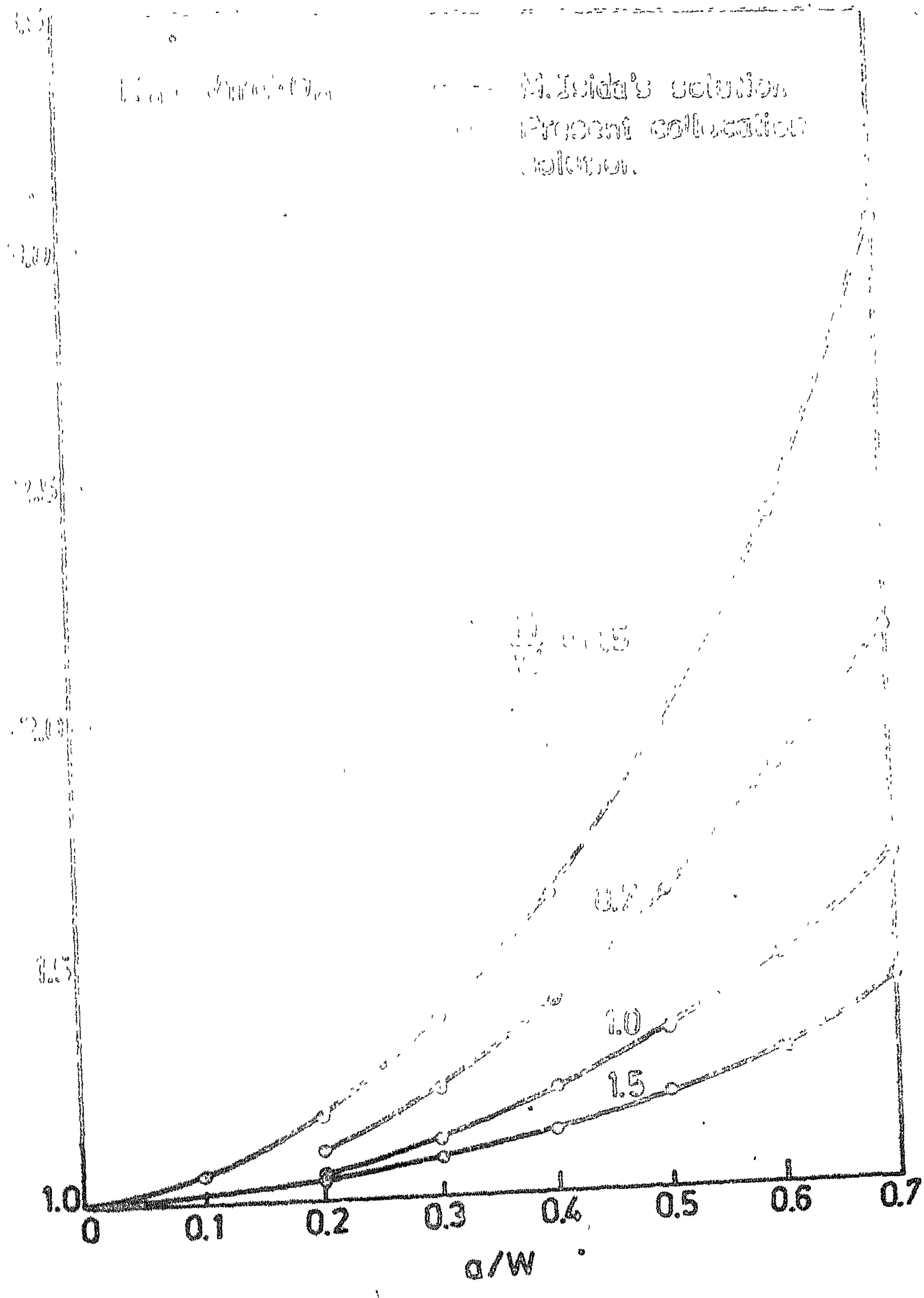


Fig. 3.3 Centre crack Problem.

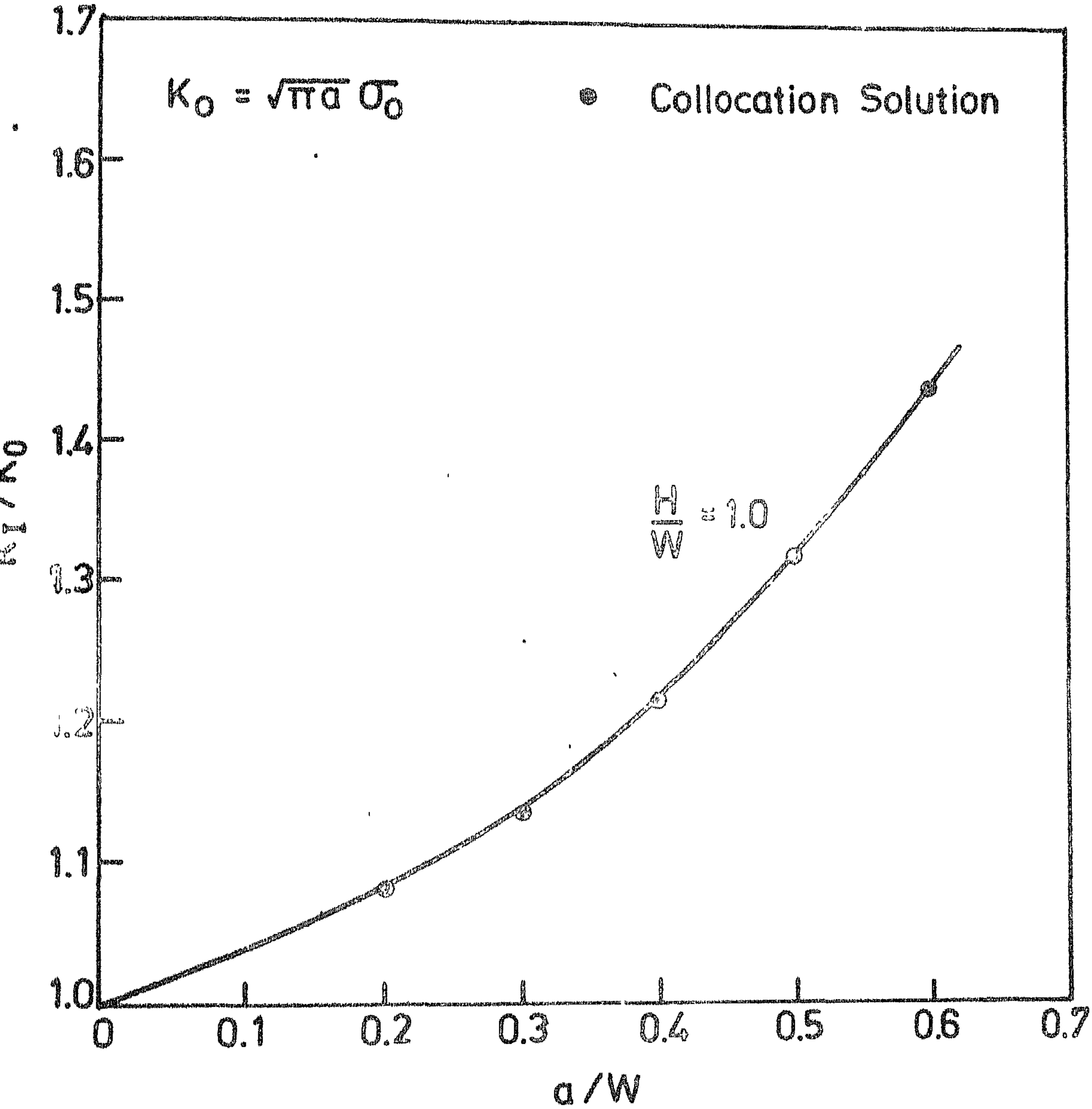


Fig. 3.4 Butt weld improper penetration problem.

TABLE-3c

Solution for single edge notch specimens with displacement prescribed boundary conditions

$H/W = 0.5 ; \quad V_o = 100.0$

| a/W | No. of boundary points 'n' | No. of constants 'k' | $d_1$ |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 0.2 | 30 | 30 | - 30.13 |
| | 33 | 30 | - 29.82 |
| 0.3 | 30 | 35 | - 29.88 |
| | 33 | 35 | - 30.10 |
| 0.4 | 30 | 30 | - 30.618 |
| | 33 | 30 | - 30.519 |
| | 30 | 40 | - 31.566 |
| | 33 | 40 | - 30.850 |
| 0.5 | 30 | 35 | - 31.520 |
| | 33 | 35 | - 31.874 |
| 0.6 | 30 | 36 | - 32.21 |
| | 33 | 36 | - 32.34 |
| 0.7 | 30 | 35 | - 32.93 |
| | 33 | 35 | - 32.89 |

Contd....Table-3c

H/W = 0.75

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 0.2 | 30 | 39 | - 26.816 |
| | 33 | 39 | - 26.886 |
| 0.3 | 30 | 39 | - 28.058 |
| | 33 | 39 | - 28.112 |
| 0.4 | 30 | 30 | - 28.917 |
| | 33 | 39 | - 28.956 |
| 0.5 | 30 | 39 | - 29.558 |
| | 33 | 39 | - 29.570 |
| 0.6 | 30 | 39 | - 29.743 |
| | 33 | 39 | - 29.668 |
| 0.7 | 30 | 39 | - 29.726 |
| | 33 | 39 | - 29.534 |
| 0.8 | 30 | 39 | - 31.133 |
| | 33 | 39 | - 29.891 |
| | 30 | 35 | - 30.992 |
| | 33 | 35 | - 30.425 |

Contd....

Contd....Table-3c

H/W = 1.0

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 0.2 | 30 | 35 | - 23.035 |
| | 33 | 35 | - 23.173 |
| 0.3 | 30 | 35 | - 24.985 |
| | 33 | 35 | - 25.144 |
| | 40 | 49 | - 25.177 |
| | 43 | 49 | - 25.326 |
| 0.4 | 30 | 39 | - 26.055 |
| | 33 | 39 | - 26.212 |
| 0.5 | 30 | 39 | - 26.700 |
| | 33 | 39 | - 26.834 |
| 0.6 | 30 | 39 | - 26.951 |
| | 33 | 39 | - 27.050 |
| 0.7 | 30 | 39 | - 27.114 |
| | 33 | 39 | - 27.197 |
| 0.8 | 30 | 39 | - 27.987 |
| | 33 | 39 | - 27.836 |
| 0.9 | 30 | 39 | - 28.959 |
| | 33 | 39 | - 28.534 |

TABLE-3d

Solution for centre crack problem with traction prescribed boundary conditions

$\sigma_o$ = 100.0 ksc. ; H/W = 0.5

| a/W (1) | No. of boundary points, n (2) | No. of constants 'k' (3) | $d_1$ (4) |
|---|---|---|---|
| 0.1 | 30 | 50 | - 23.822 |
| | 33 | 50 | - 23.692 |
| 0.2 | 30 | 48 | - 37.151 |
| | 33 | 48 | - 37.128 |
| 0.3 | 30 | 48 | - 53.092 |
| | 33 | 48 | - 53.091 |
| 0.4 | 30 | 50 | - 72.872 |
| | 33 | 50 | - 72.868 |
| 0.5 | 30 | 50 | - 98.375 |
| | 33 | 50 | - 98.361 |
| 0.6 | 30 | 50 | -132.826 |
| | 33 | 50 | -132.787 |
| 0.7 | 30 | 50 | -179.699 |
| | 33 | 50 | -179.799 |
| 0.8 | 30 | 50 | -217.2755 |
| | 33 | 50 | -223.683 |
| | 30 | 20 | -243.53 |
| | 33 | 20 | -230.50 |

Contd....

Contd.....Table-3d

H/W = 0.7 , $\sigma_o$ = 100.0 ksc

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| | 25 | 40 | - 23.921 |
| 0.1 | 28 | 40 | - 23.892 |
| | 35 | 60 | - 24.041 |
| | 38 | 60 | - 24.048 |
| 0.2 | 35 | 60 | - 34.875 |
| | 38 | 60 | - 34.874 |
| 0.3 | 35 | 60 | - 47.554 |
| | 60 | 60 | - 47.555 |
| 0.4 | 35 | 60 | - 62.601 |
| | 38 | 60 | - 62.602 |
| 0.5 | 35 | 60 | - 80.941 |
| | 38 | 60 | - 80.940 |
| 0.6 | 35 | 60 | -103.164 |
| | 38 | 60 | -103.161 |
| | 35 | 60 | -129.483 |
| | 38 | 60 | -129.896 |
| 0.7 | 40 | 64 | -126.761 |
| | 43 | 64 | -129.202 |

Contd....

Contd.....Table 3d

H/W = 1.0 , $\sigma_o$ = 100.0 ksc

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| 0.1 | 30 | 50 | - 23.832 |
| | 33 | 50 | - 24.059 |
| 0.2 | 30 | 50 | - 33.487 |
| | 33 | 50 | - 33.581 |
| | 35 | 50 | - 33.456 |
| | 38 | 50 | - 33.426 |
| | 35 | 60 | - 33.042 |
| | 38 | 60 | - 33.570 |
| 0.3 | 30 | 48 | - 43.511 |
| | 33 | 48 | - 43.516 |
| 0.4 | 30 | 48 | - 54.386 |
| | 33 | 48 | - 54.382 |
| 0.5 | 30 | 48 | - 66.687 |
| | 33 | 48 | - 66.683 |
| 0.6 | 30 | 48 | - 81.074 |
| | 33 | 48 | - 81.098 |
| 0.7 | 25 | 40 | - 98.928 |
| | 28 | 40 | - 99.102 |
| | 30 | 48 | -101.722 |
| | 33 | 48 | - 99.923 |

Contd.....

Contd.....Table-3d

H/W=1.5 ; $\sigma_o$ = 100.0 ksc

| 1 | 2 | 3 | 4 |
|---|---|---|---|
| | 35 | 40 | - 26.975 |
| | 38 | 40 | - 25.090 |
| | 45 | 30 | - 26.714 |
| 0.1 | 48 | 30 | - 26.690 |
| | 40 | 30 | - 26.688 |
| | 43 | 30 | - 26.730 |
| | 40 | 40 | - 33.061 |
| 0.2 | 43 | 40 | - 30.561 |
| | 40 | 30 | - 34.209 |
| | 43 | 30 | - 34.210 |
| 0.3 | 40 | 30 | - 41.939 |
| | 43 | 30 | - 41.929 |
| 0.4 | 40 | 30 | - 50.404 |
| | 43 | 30 | - 50.380 |
| | 40 | 30 | - 60.071 |
| 0.5 | 43 | 30 | - 60.035 |
| | 40 | 40 | - 60.060 |
| | 43 | 40 | - 60.075 |
| 0.6 | 40 | 40 | - 70.089 |
| | 43 | 40 | - 70.307 |
| | 40 | 40 | - 82.786 |
| | 43 | 40 | - 92.564 |
| 0.7 | 40 | 30 | - 84.514 |
| | 43 | 30 | - 84.587 |
| | 40 | 34 | - 86.224 |
| | 43 | 34 | - 86.173 |
| | 40 | 36 | - 86.907 |

## TABLE-3e

Solution for Butt weld improper penetration problem

W = 0.5 inch

R = 1.04 inch

C = 0.3 inch

| a/w | No. of boundary points 'n' | No. of constants 'k' | first coefficient, $d_1$ |
|---|---|---|---|
| 0.2 | 25 | 40 | -24.208 |
| | 28 | 40 | -24.180 |
| 0.3 | 25 | 40 | -30.998 |
| | 28 | 40 | -30.991 |
| 0.4 | 25 | 40 | -38.410 |
| | 28 | 40 | -38.410 |
| 0.5 | 25 | 40 | -46.571 |
| | 28 | 40 | -46.572 |
| | 25 | 40 | -55.709 |
| | 28 | 40 | -55.703 |

finite element method for this range of solution is 4.5%. The solution of this problem is plotted in fig. (2.2) with finite element solution.

The centre crack problem under tensile loading as discussed in (section 2.8) is analysed and the results (fig. 3.3) compared well with those obtained by M.Isida [2]. M. Isida used a complex stress function prescribed for centre cracks using the stress function and its normal derivative as the boundary conditions. The good agreement of our results with those of Isida's gives us confidence to proceed further with the problem of butt-weld partial penetration. The result of this problem for a sample input of H/W ratio equal to 1.0 is plotted in fig. (3.4) in the normalized form. The solution of stress intensity factor can be used to find the stress concentration factor, by assuming a fictitious crack tip radius as follows:

$$K_I = \frac{1}{2} K_N . \sigma . \sqrt{\pi \rho} \qquad (3.1)$$

where $K_N$ is the elastic stress concentration factor and '$\sigma$' is the far field stress from the crack tip.
$\rho$ is the fictitious crack tip radius
The stress intensity factor $K_I$ can also be used to study the fatigue growth of the cracks and permissible defect sizes can also be predicted.

3.3 Conclusions:

From our present work on Collocation procedure to find stress intensity factors for both single edge crack and centre crack problems the following conclusions can be made.

1) Collocation method with boundary conditions involving second order partial differentiation of the stress function give good results for the problems discussed in our work,if the number of boundary points taken is more than the number of arbitrary contants of the William's stress function.

2) Collocation procedure offers the distinct advantage of easy input preparation. For different a/W and H/W ratios, solutions are obtained without tedious input preparations.

3) The usage of William's stress function is extended to centre crack problems as a step to generalize the procedure.

4) The convergence of the coefficient, $d_1$ is good, as the number of points over the boundary and the number of constants of the stress function are increased even if a part of geometrical boundary is curved (e.g. weld specimen).

5) This method can be used to optimize the specimen size for fracture toughness testing purposes. For example in the case of single edge notched specimen as there is no appreciable change in stress intensity factor for ratio of H/W above 0.75 for a/W ratios 0.2 (fig. 3.1) the specimen size can be limited with H/W ratio as 0.75.

6) The easy method of finding stress intensity factor for weld specimens enables one to carry out studies on fatigue growth of cracks and to prescribe allowable defect sizes of weld geometries.

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